Chapter 1 Real numbers

1. Euclid's Division lemma:- Given Positive integers a and b there exist unique integers q and r satisfying

a=bq +r, where 0 $\leq$r<b,where a, b, q and r are respectively called as dividend, divisor, quotient and remainder.

2. Euclid's division Algorithm:- To obtain the HCF of two positive integers say c and d, with c>0, follow the steps below:

Step I: Apply Euclid's division lemma, to c and d, so we find whole numbers, q and r such that c =dq+r, 0$\leq r < d$

Step II: If r=0, d is the HCF of c and d. If $\neq 0, apply\ the$ division lemma to d and r.

Step III: Continue the process till the remainder is zero. The divisor at this stage will be the required HC

3. The Fundamental theorem of Arithmetic:-

Every composite number can be expressed (factorised) as a product of primes, and this factorization is unique, apart from the order in which the prime factors occur.

Ex.:$24 = 2\ X\ 2\ X\ 2\ X\ 3 = 3\ X\ 2\ X\ 2\ X\ 2$

Theorem: LETx be a rational number whose decimal expansion terminates. Ther$x$ can be expressed in the form

Of $\frac{p}{q}$ where $p\ and\ q$ are co-prime and the prime factorisation of q is the form of $2^n.5^m$,

where n, m are non negative integers.

Ex. $\frac{7}{10} = \frac{7}{2\ X\ 5} = 0.7$

1. If the H C F of 657 and 963 is expressible in the form of 657x + 963x - 15 find x.Definition (Ans:x=22)

Ans: Using Euclid's Division Lemma

a= bq+r , o $\leq$ r < b

963=657×1+306

657=306×2+45

306=45×6+36

45=36×1+9

36=9×4+0

∴ HCF (657, 963) = 9

now 9 = 657x + 963× (-15)

657x=9+963×15

=9+14445

657x=14454

x=14454/657

x =22

2. Express the GCD of 48 and 18 as a linear combination. (Ans: Not unique)

A=bq+r, where o $\leq$ r < b

48=18x2+12

18=12x1+6

12=6x2+0

∴ HCF (18,48) = 6

now 6= 18-12x1

6= 18-(48-18x2)

6= 18-48x1+18x2

6= 18x3-48x1

6= 18x3+48x(-1)

i.e. 6= 18x +48y

∴ **x=3 , y=-1**

$6 = 18\times3 + 48\times(-1)$
$= 18\times3 + 48\times(-1) + 18\times48 - 18\times48$
$= 18(3+48) + 48(-1-18)$
$= 18\times51 + 48\times(-19)$
$6 = 18x + 48y$

$\therefore$ $\boxed{x = 51,\ y = -19}$

Hence, x and y are not unique.

3. Prove that one of every three consecutive integers is divisible by 3.

Ans:

n,n+1,n+2 be three consecutive positive integers

We know that n is of the form 3q, 3q +1, 3q + 2

So we have the following cases Properties

Case – I when n = 3q

In the this case, n is divisible by 3 but n + 1 and n + 2 are not divisible by 3

Case - II When n = 3q + 1
Sub n = 2 = 3q +1 +2 = 3(q +1) is divisible by 3. but n and n+1 are not divisible by 3

Case – III When n = 3q +2
Sub n = 2 = 3q +1 +2 = 3(q +1) is divisible by 3. but n and n+1 are not divisible by 3

Hence one of n, n + 1 and n + 2 is divisible by 3

4. Find the largest possible positive integer that will divide 398, 436, and 542 leaving remainder 7, 11, 15 respectively.

(Ans: 17)

Ans: The required number is the HCF of the numbers

Find the HCF of 391, 425 and 527 by Euclid's algorithm

$\therefore$ HCF (425, 391) = 17

Now we have to find the HCF of 17 and 527
$527 = 17 \times 31 + 0$
$\therefore$ HCF (17,527) = 17
$\therefore$ HCF (391, 425 and 527) = 17

5. Find the least number that is divisible by all numbers between 1 and 10 (both inclusive).
(Ans:2520)

Ans: The required number is the LCM of 1,2,3,4,5,6,7,8,9,10

$\therefore \text{LCM} = 2 \times 2 \times 3 \times 2 \times 3 \times 5 \times 7 = 2520$

6. Show that 571 is a prime number.

Ans: Let $x = 571 \Rightarrow \sqrt{x} = \sqrt{571}$

Now 571 lies between the perfect squares of $(23)^2$ and $(24)^2$
Prime numbers less than 24 are 2,3,5,7,11,13,17,19,23
Since 571 is not divisible by any of the above numbers
571 is a prime number

7. If d is the HCF of 30, 72, find the value of x & y satisfying d = 30x + 72y.

Ans: Using Euclid's algorithm, the HCF (30, 72)

$72 = 30 \times 2 + 12$
$30 = 12 \times 2 + 6$
$12 = 6 \times 2 + 0$

HCF (30,72) = 6
$6 = 30 - 12\times2$
$6 = 30 - (72 - 30\times2)2$
$6 = 30 - 2\times72 + 30\times4$
$6 = 30\times5 + 72\times-2$
$\therefore x = 5, y = -2$

Also $6 = 30 \times 5 + 72(-2) + 30 \times 72 - 30 \times 72$

Solve it, to get

$x = 77, y = -32$

Hence, x and y are not unique

8. Show that the product of 3 consecutive positive integers is divisible by 6.

Ans: Proceed as in question sum no. 3

9. Show that for odd positive integer to be a perfect square, it should be of the form
8k +1.

Let a=2m+1

Ans: Squaring both sides we get

$a^2 = 4m(m+1) + 1$

$\therefore$ product of two consecutive numbers is always even

$m(m+1)=2k$

$a^2=4(2k)+1$

$a^2 = 8k+1$

Hence proved

10. Find the greatest number of 6 digits exactly divisible by 24, 15 and :

(Ans:999720)

Ans: LCM of 24, 15, 36

LCM = $3 \times 2 \times 2 \times 2 \times 3 \times 5 = 360$

Now, the greatest six digit number is 999999

Divide 999999 by 360

$\therefore$ Q = 2777 , R = 279

$\therefore$ the required number = 999999 – 279 = 999720

11. If a and b are positive integers. Show tha $\sqrt{2}$ always lies between $\frac{a}{b}$ and $\frac{a-2b}{a+b}$

Ans: We do not know whether $\frac{a^2-2b^2}{b(a+b)}$ or $\frac{a}{b} < \frac{a+2b}{a+b}$

$\therefore$ to compare these two number,

Let us comute $\frac{a}{b} - \frac{a+2b}{a+b}$

=> on simplifying , we get $\frac{a^2-2b^2}{b(a+b)}$

$\therefore \frac{a}{b} - \frac{a+2b}{a+b} > 0$ or $\frac{a}{b} - \frac{a+2b}{a+b} < 0$

now $\frac{a}{b} - \frac{a+2b}{a+b} > 0$

$\frac{a^2-2b^2}{b(a+b)} > 0$ solve it , we get , $a > \sqrt{2}b$

Thus , when $a > \sqrt{2}b$ and

$\frac{a}{b} < \frac{a+2b}{a+b}$,

We have to prove that $\frac{a+2b}{a+b} < \sqrt{2} < \frac{a}{b}$

Now a $> \sqrt{2}$ b $\Rightarrow 2a^2+2b^2 > 2b^2 + a^2 + 2b^2$

On simplifying we get

$\sqrt{2} > \frac{a+2b}{a+b}$

Also $a > \sqrt{2}$

$\Rightarrow \frac{a}{b} > \sqrt{2}$

Similarly we get $\sqrt{2}, < \frac{a+2b}{a+b}$

Hence $\frac{a}{b} < \sqrt{2} < \frac{a+2b}{a+b}$

12. Prove that $(\sqrt{n-1} + \sqrt{n+1})$ is irrational, for every n$\in$N Symbol chart